
DTU

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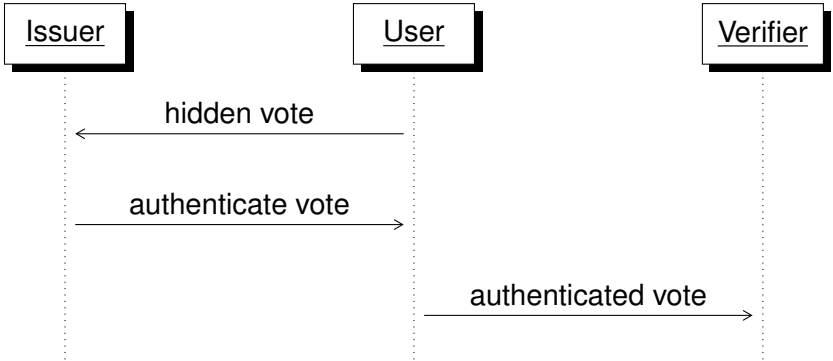
Blind Signatures Based on VOLEitH and MAYO

Example: E-Voting with Blind Signatures



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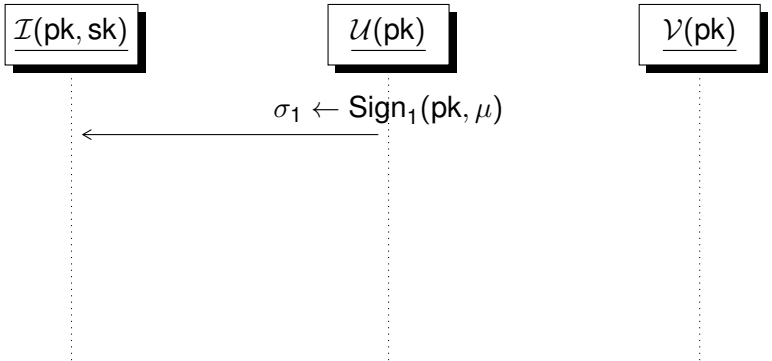
Blind Signatures



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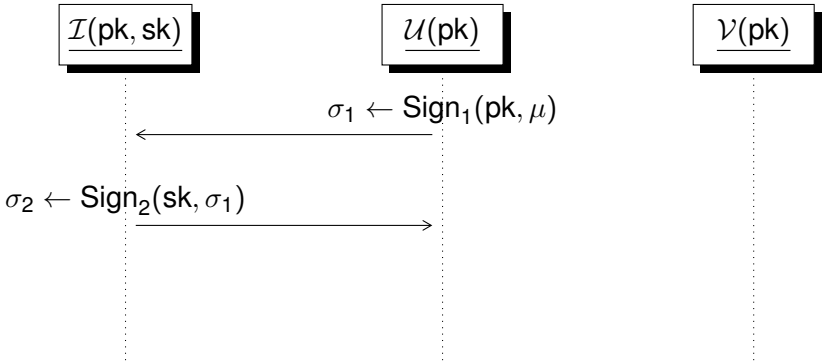

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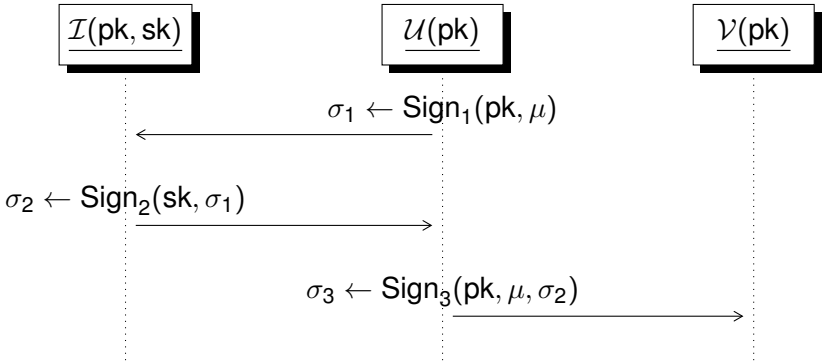

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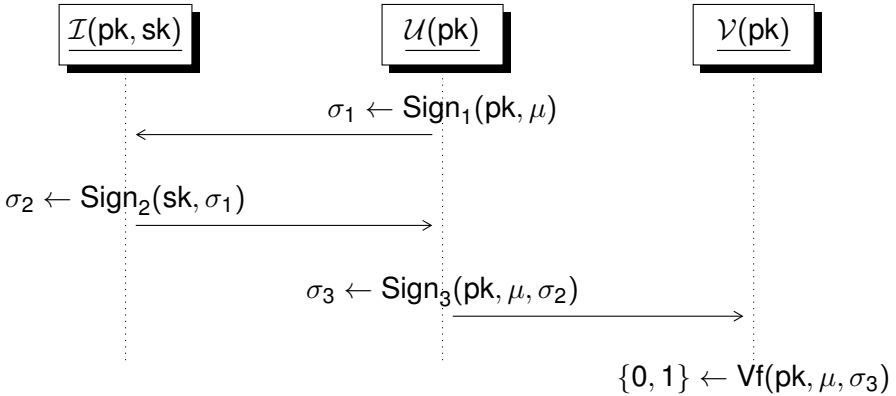
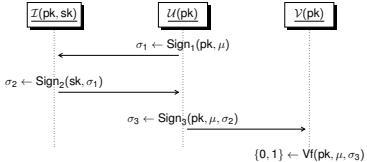


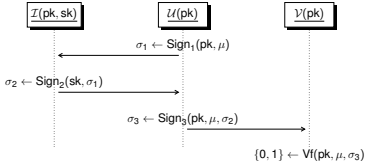
Blindness Property of Blind Signatures

- A cooperating issuer and verifier can associate the first message with the final signature



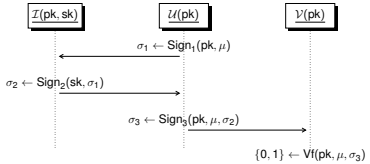
Blindness Property of Blind Signatures

- A cooperating issuer and verifier can associate the first message with the final signature
- $\mathcal{A}$ takes the role of $\mathcal{I}$ and $\mathcal{V}$ and for m_0, m_1, and $\mathcal{A}$ can not associate $\sigma_{1,m_0/m_1}$ with respective final signatures $\sigma_{3,m_0/m_1}$



Fischlin's Blueprint for Blind Signatures

- $(\mathsf{pk}, \mathsf{sk})$ are from a normal signature scheme



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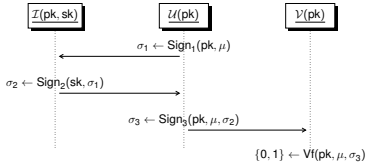
- $\sigma_1 \leftarrow \mathit{Com}(\mu, r)$ where $r \leftarrow \{0,1\}^{1^\lambda}$



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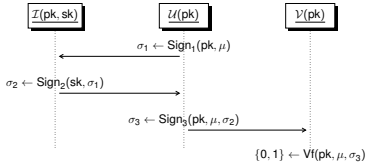

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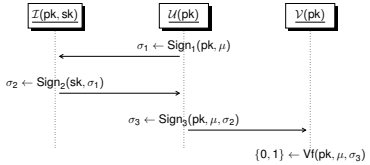
- $\sigma_1 \leftarrow \mathit{Com}(\mu, r)$ where $r \leftarrow \{0,1\}^{1^\lambda}$
- $\sigma_2 \leftarrow \mathsf{Sign}(\mathsf{sk}, \sigma_1)$
- $\sigma_3 \leftarrow (\mu, z)$ where $z \leftarrow \mathit{ZKP}(\mathsf{pk}, \sigma_2, \mu, r)$



High-Level Description of our Blind Signature

For a fixed μ, and fixed MAYO keys $(\mathsf{pk}, \mathsf{sk})$

1 : $r \leftarrow\!\!\$\ \{0,1\}^{\lambda}$

2 : $c \leftarrow \text{SHAKE256}(r \| \mu)$

3 : $salt \leftarrow\!\!\$\ \{0,1\}^{\lambda}$

4 : $t \leftarrow \text{SHAKE256}(salt \| c)$

5 : $\sigma \leftarrow \text{MAYO.SamplePre}(\mathsf{sk}, \mathsf{pk}, t)$

6 : $\text{VOLEitH-ZKP}(\mu, r, salt, \sigma)$

Vector Oblivious Linear Evaluation (VOLE)

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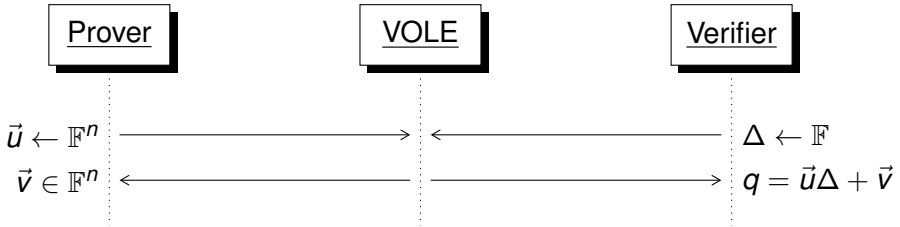
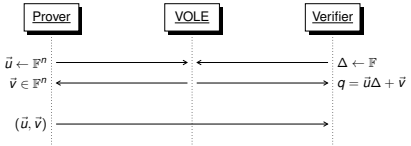


▷ The verifier evaluates the polynomial $p(x) = \vec{u}x + \vec{v}$ at position Δ.

View VOLE as a Commitment Scheme to $\vec{u}$

- Prover can send $(\vec{u}, \vec{v})$ to open



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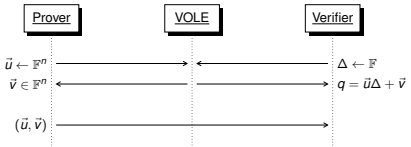

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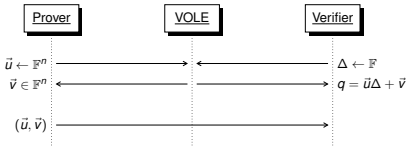

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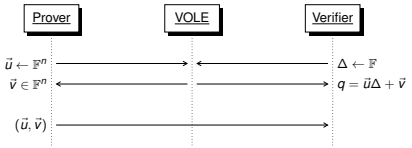
- *Binding*: opening to $u' \neq u$ requires guessing Δ $(u'\Delta + \vec{v}' = u\Delta + \vec{v} = q \Rightarrow \Delta = (\vec{v} - \vec{v}')/(\vec{u}' - \vec{u}))$



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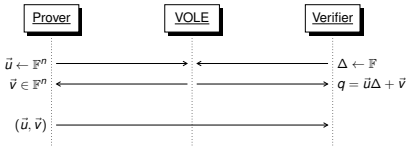
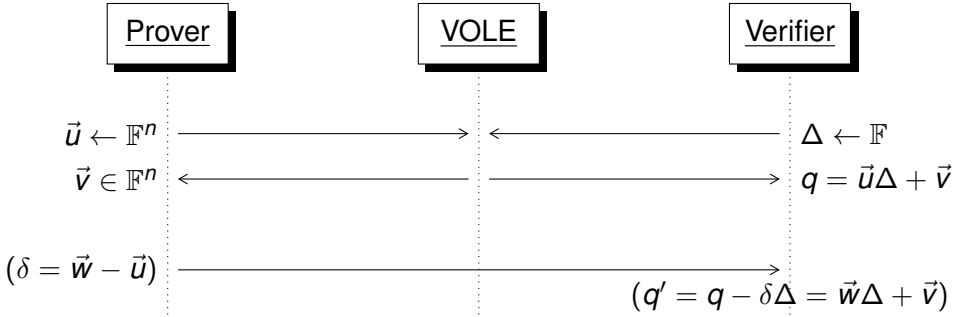
- Evaluable as a circuit with n multiplications and n additions

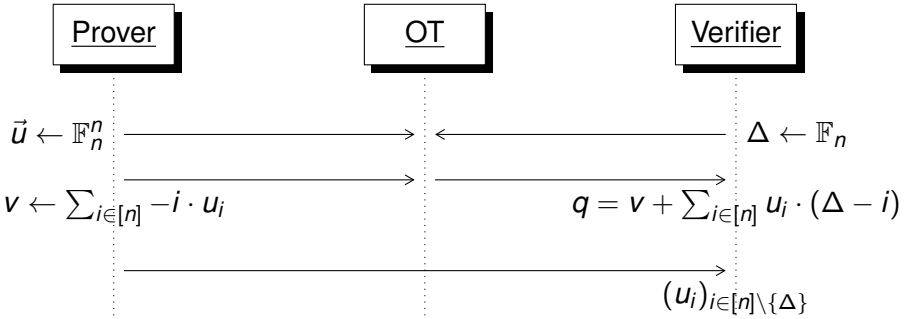


Cast Commitment Scheme to a Witness $\vec{w}$



Casting VOLE to get Public Verifiability

- Generate VOLE from a n-out-of-n secret sharing scheme and OT
- Δ now only determines which $n-1$ challenges need to be opened



Casting VOLE to get Public Verifiability

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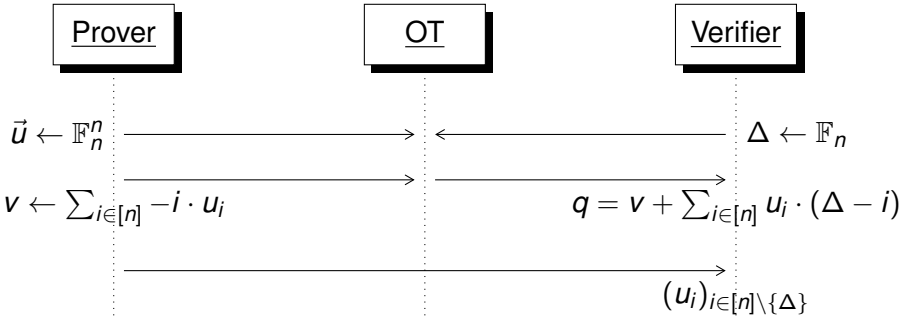
- Derive Δ using hash of commitments and cast in Fiat-Shamir style



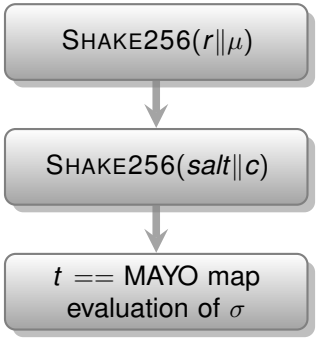
Properties of VOLEitH as a Commit&Proof Framework

- linear: adding two commitments gives commitment to sum of witnesses
- multiplication: multiplying gives you a degree increase
- degree decrease: at the cost of n additional commitments
- zero-check

Our ZKP using VOLEitH

ZKP Strategy

1. bit-commit to $r \| \mu$
2. SHAKE256 with bit-commit and VOLEitH properties
3. SHAKE256 with previous result and bit-commitment to salt
4. $\mathbb{F}_{16}$ commitments of σ
5. MAYO map evaluation with VOLEitH properties
6. Comparison using 0-check



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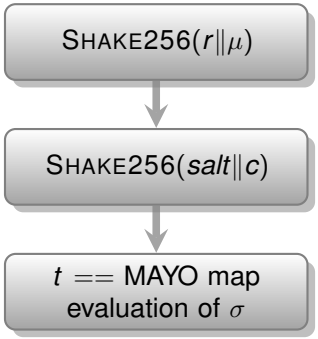
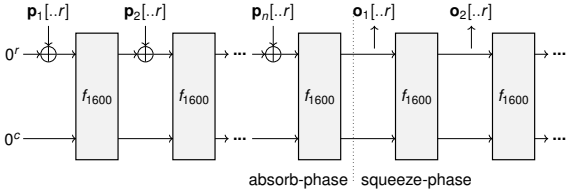
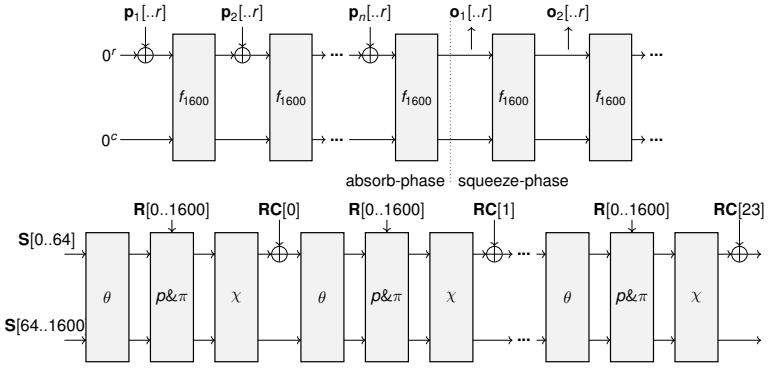


SHAKE256


$\mathbf{p}_1[..r]$
$\mathbf{p}_2[..r]$
$\mathbf{p}_n[..r]$
$\mathbf{o}_1[..r]$
$\mathbf{o}_2[..r]$
0^r
0^c
f_{1600}
absorb-phase
squeeze-phase

SHAKE256


$\mathbf{p}_1[..r]$
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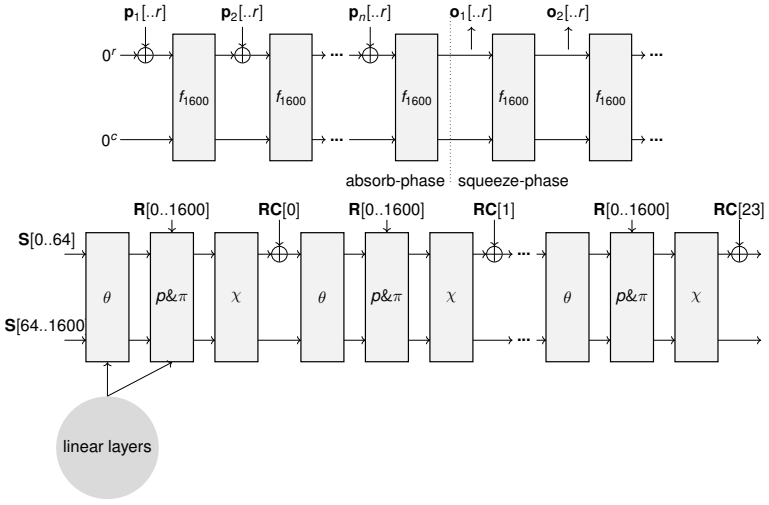
$\mathbf{o}_1[..r]$
$\mathbf{o}_2[..r]$
0^r
0^c
f_{1600}
absorb-phase
squeeze-phase
$\mathbf{R}[0..1600]$
$\mathbf{RC}[0]$
$\mathbf{RC}[1]$
$\mathbf{RC}[23]$
$\mathbf{S}[0..64]$
$\mathbf{S}[64..1600]$
θ
$p\&\pi$
χ

SHAKE256


p1[..r]
p2[..r]
pn[..r]
o1[..r]
o2[..r]
0^r
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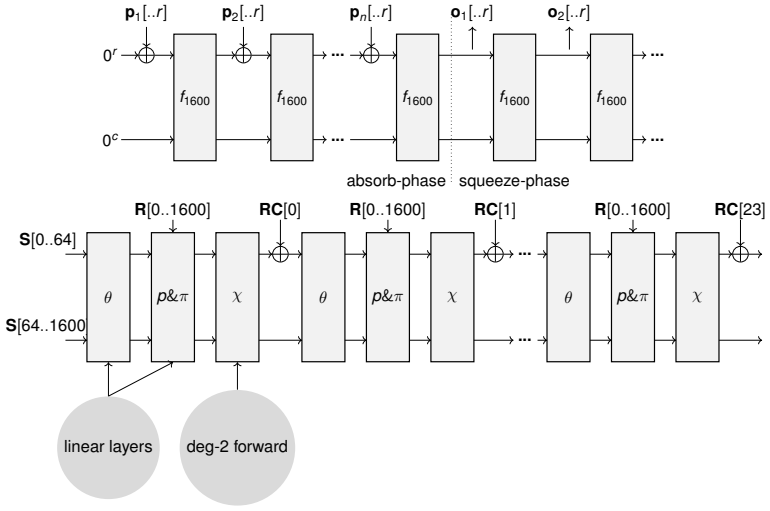
f1600
absorb-phase
squeeze-phase
R[0..1600]
RC[0]
RC[1]
RC[23]
S[0..64]
S[64..1600]
θ
p&π
χ
linear layers

SHAKE256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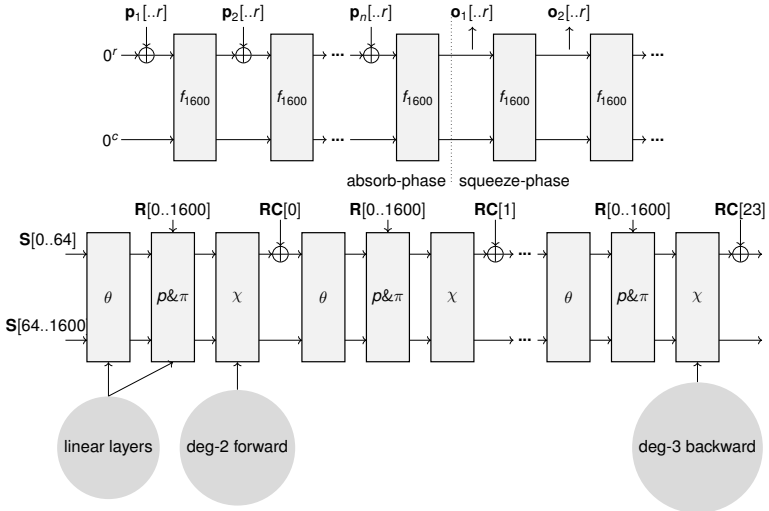
linear layers
deg-2 forward

SHAKE256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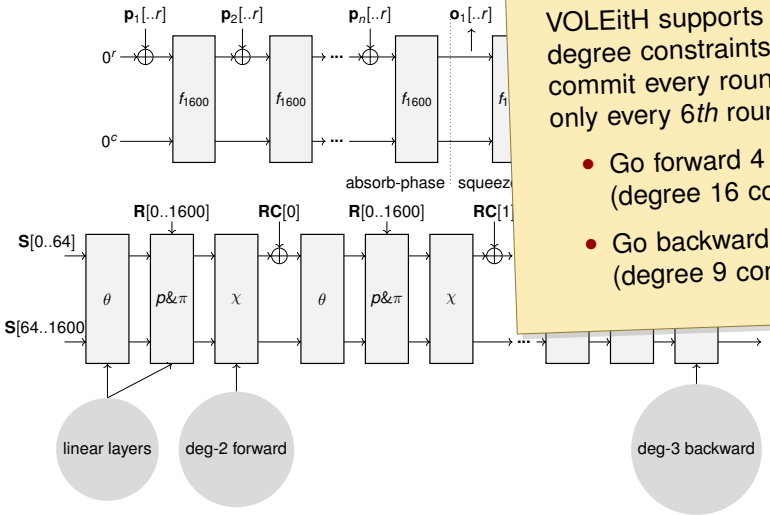

Optimization Strategy:

VOLEitH supports higher-degree constraints: Do not commit every round, but only every 6*th* round.

- Go forward 4 steps (degree 16 constraints)
- Go backward 2 steps (degree 9 constraints)

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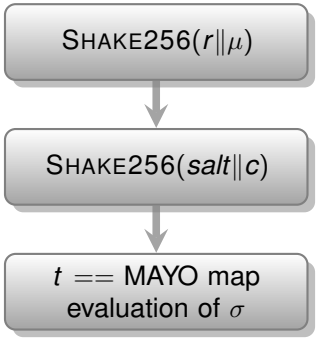

MAYO Map Evaluation

- There are n public matrices $\boldsymbol{M}_1, ..., \boldsymbol{M}_n \in \mathbb{F}_q^{m \times m}$
- The signature $\vec{s}$ is generated such that

$$\forall i \in [n] : v_i = \vec{s}^\top \boldsymbol{M}_i \vec{s}$$

where $H(\mu) = \vec{v} \in \mathbb{F}_q^n$

▷ Verifying the signature is only a degree-2 operation on the signature $\vec{s}$

Competitiveness

Scheme	Assumptions	Signing (in KB)	Showing (in KB)	Runtime Showing (in ms)	Security (in bits)
Agrawal et al. [AKSY22]	One-More-ISIS	1.4	45	–	109
del Pino[a] & Katsumata [dPK22]	MSIS/MLWE/NTRU	851	102.6	–	128
Lyubashevsky[b] et al. [LNP22]	MSIS/MLWE	$\gg 1000$	150	–	128
Bootle et al. [BLNS23b]	ISIS-f/NTRU	> 100	> 100	–	128
Policharla et al. [PWFW23]	MSIS/MLWE+Poseidon	2.4	$85 - 173$	$304 - 4822$	115
Lyubashevsky[c] et al. [LSS24]	ISIS-f/NTRU	1.3	29	< 200	128
Argo et al. [AGJ+24] (implementing [JRS23])	MSIS/MLWE	45	79.6	< 500	128
Beullens[d] et al. [BLNS23a]	MSIS/MLWE/NTRU	60	22	–	128
Jeudy and Sanders [JS24]	MSIS/MLWE	60	41	–	128
Baldimtsi[e] et al. [BCGY24]	One-More-ISIS	1	68	–	128
SHAKE256-deg16+MAYO-$128_{s/f}$	UOV/WMQ	0.486	24.3/41.6	178.1/75.6	128
RainHash+MAYO-$128_{s/f}$	UOV/WMQ	0.486	16.5/27.8	114.4/49.8	128
One-More-MAYO-$128_{s/f}$	UOV/One-More-WMQ	0.469	6.7/10.4	42/40	128

[a] May be improved upon using more recent lattice NIZKs. [b] Bounded number of signatures, 1.3MB public key. [c] Implements a version of [BLNS23b].
[d] Uses a generic NIZK scheme. [e] Non-interactive construction with pseudorandom messages.

Competitiveness

2 additional constructions:

RainHash+MAYO-128$_{s/f}$ Uses RAINHASH instead of SHAKE256, which is friendlier for ZKPs.

One-More-MAYO-128$_{s/f}$ Uses stronger assumption on underlying signature, but does not have any hash evaluation in the blind signature

Scheme					
Agrawal et al. [AKSY22] del Pino[a] & Katsumata [dPK22] Lyubashevsky[b] et al. [LNP22] Bootle et al. [BLNS23b] Policharla et al. [PWFW23] Lyubashevsky[c] et al. [LSS24] Argo et al. [AGJ+24] (implementing [JRS23]) Beullens[d] et al. [BLNS23a] Jeudy and Sanders [JS24] Baldimtsi[e] et al. [BCGY24]					
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Thanks for listening! Paper on eprint: 2026/109

References I

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Extra Slides

Removing The Hash Calls

- Achieve the hiding not through a commitment scheme, but inherent randomness from the proof generation:

$$t = H(\mu, \pi_1) \oplus r$$

  where $(\pi_1, r) \leftarrow \mathcal{P}_1$. This only adds small overhead to the proof itself.
- Use a new assumption: One-More-MAYO, that allows querying the MAYO map for chosen targets

▷ Less conservative option, but it significantly improves runtime and size. (Primarily impacts for smaller security parameters)